DIGITAL-COMPUTER THEORETICAL INVESTIGATIONS OF NONSTATIONARY PROCESSES IN THERMOELECTRIC GENERATORS

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Equations are given for the nonstationary processes occurring in thermoelectric generators; a method is developed for investigating them by digital computer. Sample calculations are given for nonstationary processes in a thermoelectric generator.

There is no exact analytic way of solving the system of differential equations that describe nonstationary processes in thermoelectric generators. Thus it is necessary to use a numerical method developed for parabolic-type systems, i.e., the method of nets or the method of straight lines [1].

Comparing these methods for our problem, we see that it is better to use the straight-line method, since the method of nets requires considerably more programing effort, and the solution is cumbersome.

By using the straight-line method to solve the system of equations describing nonstationary processes in thermoelectric generators, we can reduce the problem to a system of ordinary first-order differential equations. This in turn makes it possible to employ standard Runge–Kutta routines.

Figure 1a shows the design scheme for the thermogenerator, which consists of flat thermocouples; the equation system for this arrangement can be written as

$$\frac{\partial u_i}{\partial t} = \mathrm{X}_i^2 \frac{\partial^2 u_i}{\partial x^2} \quad (i = 1,\ 2,\ 4,\ 6,\ 7), \quad x_{i-1} < x < x_i;$$
$$\frac{\partial u_3}{\partial t} = \mathrm{X}_3^2 \frac{\partial^2 u_3}{\partial x^2} - \frac{1}{2}\vartheta_J, \quad x_2 < x < x_3; \tag{1}$$
$$\frac{\partial u_5}{\partial t} = \mathrm{X}_5^2 \frac{\partial^2 u_5}{\partial x^2} + \frac{1}{2}\vartheta_J, \quad x_4 < x < x_5$$

with the initial and boundary conditions

$$u_i(x)|_{t=0} = \Psi_i(x) \quad (i = 1,\ 2,\ \ldots,\ 7);$$
$$\left(\frac{\partial u_1}{\partial x} + h_1 u_1\right)_{x=x_0} = h_1 T_{\text{med}} \tag{2}$$

$$\left(-\frac{\partial u_7}{\partial x} + h_7 u_7\right)_{x=x_7} = h_7 T_{\text{c}} \tag{3}$$

and the contact conditions

$$\lambda_{T_i}\frac{\partial u_i}{\partial x} = \lambda_{T_{i+1}}\frac{\partial u_{i+1}}{\partial x}, \quad u_i = u_{i+1}, \quad x = x_i$$
$$(i = 1,\ 2,\ 5,\ 6);$$

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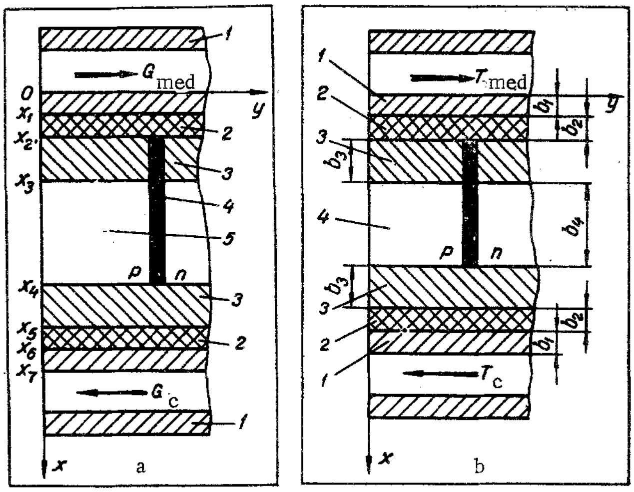

Fig. 1. Design (a) and structural (b) diagrams for thermoelectric generator: a) 1) heat line on hot-junction side; 2) electrical insulation; 3) connecting plates; 4) thermal insulation; 5) semiconductor material; b) 1) steel; 2) mica; 3) copper; 4) semiconductor material.

$$\lambda_{T_3} \frac{\partial u_3}{\partial x} = \lambda_{T_4} \frac{\partial u_4}{\partial x} + q_{\text{ph}}, \quad u_3 = u_4, \quad x = x_3;$$
$$\lambda_{T_4} \frac{\partial u_4}{\partial x} = \lambda_{T_5} \frac{\partial u_5}{\partial x} + q_{\text{pc}}, \quad u_4 = u_5, \quad x = x_4, \tag{4}$$

where $u_i = u(x, t)$ $(i = 1, \ldots, 7)$ is a continuous function of the temperature distribution, which has piecewise-continuous derivatives up to order two inclusive.

We write the system of equations for the thermogenerator [2] as

$$I = \frac{\alpha_s [u_4(x_3, t) - u_4(x_4, t)]}{(M+1) r};$$
$$U = k_{\text{ser}} \ \alpha_s [u_4(x_3, t) - u_4(x_4, t)] - I k_{\text{ser}} \, r; \tag{5}$$
$$P = I^2 M k_{\text{ser}} \, r;$$
$$\vartheta_J = \frac{I^2 k_{\text{ser}} \, r}{cV\delta}, \quad q_{\text{ph}} = \frac{\alpha_s I u_4(x_3, t)}{F}, \quad q_{\text{pc}} = -\frac{\alpha_s I u_4(x_4, t)}{F}.$$

Next, to simplify (1), we make the approximate substitution

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{x=x_j} = \frac{1}{h^2} [u(x_{j+1}, t) - 2u(x_j, t) + u(x_{j-1}, t)], \tag{6}$$

while we use the following formulas for the contact and boundary conditions:

$$\left.\frac{\partial u}{\partial x}\right|_{x=x_j} = \frac{1}{h} [u(x_j, t) - u(x_{j-1}, t)] \quad (j = 1, \ldots, n-1) \tag{7}$$

and also let

$$u(x_j, t) = u_j(t), \quad \frac{\partial u(x_j, t)}{\partial t} = \dot{u}_j(t) \quad (j = 1, \ldots, n-1).$$

Substituting (6) and (7) into (1)-(5), we obtain a system of ordinary differential equations:

$$\dot{u}_{1,i} = \frac{\mathrm{X}_1^2}{h_1^2} [u_{1,\,i+1}(t) - 2u_{1,\,i}(t) + u_{1,i+1}(t)] \quad (i = 1, \ldots, 9), \quad x_0 < x < x_1;$$
$$\dot{u}_{2,\,i} = \frac{\mathrm{X}_2^2}{h_2^2} [u_{2,\,i+1}(t) - 2u_{2,\,i}(t) + u_{2,i-1}(t)] \quad (i = 1, \ldots, 4), \quad x_1 < x < x_2; \tag{8}$$
$$\dot{u}_{3,\,i} = \frac{\mathrm{X}_3^2}{h_3^2} [u_{3,\,i+1}(t) - 2u_{3,\,i}(t) + u_{3,\,i-1}(t)] - \frac{1}{2} \vartheta_{J_i} \quad (i = 1, \ldots, 4), \quad x_2 < x < x_3;$$

$$\dot{u}_{4,i}=\frac{X_4^2}{h_4^2}\left[u_{4,i+1}(t)-2u_{4,i}(t)+u_{4,i-1}(t)\right]\quad (i=1,\ \ldots,\ 9),\ x_3<x<x_4;$$

$$\dot{u}_{5,i}=\frac{X_5^2}{h_5^2}\left[u_{5,i+1}(t)-2u_{5,i}(t)+u_{5,i-1}(t)\right]+\frac{1}{2}\vartheta_{J_i}\quad (i=1,\ \ldots,\ 4),\quad x_4<x<x_5;$$

$$\dot{u}_{6,i}=\frac{X_6^2}{h_6^2}\left[u_{6,i+1}(t)-2u_{6,i}(t)+u_{6,i-1}(t)\right]\quad (i=1,\ \ldots,\ 4),\quad x_5<x<x_6;$$

$$\dot{u}_{7,i}=\frac{X_7^2}{h_7^2}\left[u_{7,i+1}(t)-2u_{7,i}(t)+u_{7,i-1}(t)\right]\quad (i=1,\ \ldots,\ 9),\ x_6<x<x_7.$$

Allowing for the above, we can write the boundary conditions as

$$\begin{gathered}[u_{1,1}(t)-u_{1,0}(t)]+h_1u_{1,0}(t)=h_1T_{\text{med}}(t),\quad x=x_0;\\ -[u_{7,10}(t)-u_{7,9}(t)]+h_7u_{7,10}(t)=h_7T_{\text{c}}\quad(t),\quad x=x_7,\end{gathered}\tag{9}$$

and the layer contact conditions as

$$\begin{gathered}\frac{\lambda_{T_1}}{h_1}[u_{1,10}(t)-u_{1,9}(t)]=\frac{\lambda_{T_2}}{h_2}u_{2,1}(t)-u_{2,0}(t)],\quad x=x_1;\\ u_{1,10}(t)=u_{2,0}(t);\\ \frac{\lambda_{T_2}}{h_2}[u_{2,5}(t)-u_{2,4}(t)]=\frac{\lambda_{T_3}}{h_3}[u_{3,1}(t)-u_{3,0}(t)],\quad x=x_2;\\ u_{2,5}(t)=u_{3,0}(t);\\ \frac{\lambda_{T_3}}{h_3}[u_{3,5}(t)-u_{3,4}(t)]=\frac{\lambda_{T_4}}{h_4}[u_{4,1}(t)-u_{4,0}(t)]+q_{\text{ph}}\quad x=x_3;\\ u_{3,5}(t)=u_{4,0}(t);\\ \frac{\lambda_{T_4}}{h_4}[u_{4,10}(t)-u_{4,9}(t)]=\frac{\lambda_{T_5}}{h_5}[u_{5,1}(t)-u_{5,0}(t)]+q_{\text{pc}}\quad x=x_4;\\ u_{4,10}(t)=u_{5,0}(t);\\ \frac{\lambda_{T_5}}{h_5}[u_{5,5}(t)-u_{5,4}(t)]=\frac{\lambda_{T_6}}{h_6}[u_{6,1}(t)-u_{6,0}(t)],\quad x=x_5;\\ u_{5,5}(t)=u_{6,0}(t);\\ \frac{\lambda_{T_6}}{h_6}[u_{6,5}(t)-u_{6,4}(t)]=\frac{\lambda_{T_7}}{h_7}[u_{7,1}(t)-u_{7,0}(t)],\quad x=x_6;\\ u_{6,5}(t)=u_{7,0}(t).\end{gathered}\tag{10}$$

In like manner, we can write (5) in the form

$$\begin{gathered}I=\alpha_s[u_{4,0}(t)-u_{4,10}(t)]\frac{1}{[M(\Delta u_4)+1]\,r(\Delta u_4)};\\ U=k_{\text{ser}}\ \alpha_s[u_{4,0}(t)-u_{4,10}(t)]-k_{\text{ser}}\,I(\Delta u_4,\ t)\,r(\Delta u_4);\\ P=k_{\text{ser}}\,I^2(\Delta u_4,\ t)\,M(\Delta u_4)\,r(\Delta u_4);\\ q_{\text{ph}}=\frac{\alpha_s}{F}I(\Delta u_4,\ t)\,u_{4,0}(t);\\ q_{\text{pc}}=-\frac{\alpha_s}{F}I(\Delta u_4,\ t)\,u_{4,10}(t);\\ \vartheta_{J_i}=\frac{k_{\text{ser}}}{cV\delta}I_i^2(\Delta u_4,\ t)\,r_i(\Delta u_4),\end{gathered}\tag{11}$$

where $\Delta u_4 = u_{4,0}-u_{4,10}$, $M = M(\Delta u_4)$, $r = r(\Delta u_4)$ are given in tabular or analytic form.

In addition,

$$I_i^2=\left\{\alpha_s(u_{4,i-1}-u_{4,i+1})\frac{1}{[\Psi_2(\Delta u_{4,i})+1]\,\Psi_1(\Delta u_{4,i})}\right\}^2,\tag{12}$$

where

$$\Psi_1(\Delta u_{4,i}) = r_i(\Delta u_4) = r(u_{4,\,i-1} - u_{4,\,i+1});$$
$$\Psi_2(\Delta u_{4,i}) = M_i(\Delta u_4) = M(u_{4,\,i-1} - u_{4,\,i+1}) \qquad (i = 1, \ldots, 9).$$

To determine the initial conditions for (1), we must solve the algebraic-equation system for stationary operation of the thermogenerator. The coefficients in the algebraic system can be found from (3) and (4). In this case, we obtain the nonsingular matrix.

To find the initial conditions for the system (8)-(11), we must find their values at the node points for the straight-line method.

As an example, let us look at the determination of the relaxation time for the thermogenerator (Fig. 1b) under a step disturbance in the temperature of the heat-transport medium.

The initial data are

$$b_1 = b_7 = 5\cdot 10^{-3}\ \text{m};\quad b_2 = b_6 = 0{,}1\cdot 10^{-3}\ \text{m};$$
$$b_3 = b_5 = 1{,}5\cdot 10^{-3}\ \text{m};\ b_4 = 4\cdot 10^{-3}\ \text{m};$$
$$T_{\text{med, max}} = 300\ {}^\circ\text{C};\quad T_{\text{c max}} = 50\ {}^\circ\text{C};$$
$$c_1 = 0{,}11\ \text{kcal/kg}\cdot\text{deg};\quad c_2 = 0{,}094\ \text{kcal/kg}\cdot\text{deg};$$
$$c_3 = 0{,}21\ \text{kcal/kg}\cdot\text{deg};\quad c_4 = 0{,}03\ \text{kcal/kg}\cdot\text{deg};$$
$$\gamma_1 = 7{,}88\cdot 10^3\ \text{kg/m}^3;\quad \gamma_2 = 8{,}93\cdot 10^3\ \text{kg/m}^3;\quad \gamma_3 = 3\cdot 10^3\ \text{kg/m}^3;$$
$$\gamma_4 = 7\cdot 10^3\ \text{kg/m}^3;$$
$$\lambda_{T_1} = 58{,}5\ \text{kcal/m}\cdot\text{h}\cdot\text{deg};\quad \lambda_{T_2} = 0{,}5\ \text{kcal/m}\cdot\text{h}\cdot\text{deg};$$
$$\lambda_{T_3} = 331\ \text{kcal/m}\cdot\text{h}\cdot\text{deg};\quad \lambda_{T_4} = 1{,}29\ \text{kcal/m}\cdot\text{h}\cdot\text{deg};$$
$$\alpha_{\text{tp}} = 4200\ \text{kcal/m}\cdot\text{h}\cdot\text{deg};$$
$$\alpha_{\text{tc}} = 3000\ \text{kcal/m}\cdot\text{h}\cdot\text{deg};\quad k_{\text{ser}} = 1;$$
$$\alpha_s = \alpha_{s_p} + \alpha_{s_n} = 0{,}3\cdot 10^{-3}\ \text{V/deg};$$
$$V = 400\ \text{mm}^2;\ F = 100\ \text{mm}^3.$$

A Minsk-22 computer was used to determine the thermal characteristics; a time integration step of $0.5\cdot 10^{-2}$ sec was used. The graphs of $r = r(\Delta u)$ and $M = M(\Delta u)$ employed in the computations were tabulated.

Figure 2 shows the relaxation-time results for an abrupt change in T_{med} from 100 to 150°C.

Thus for the indicated thermal-characteristic values and the given arrangement of the thermogenerator, the relaxation time is $t_r = 5$ sec. If we allow for the normally permitted range of fluctuation in voltage and current, however, this time is less than 1 sec.

In like manner, we can obtain t_r for disturbances in T_c or in T_{med} and T_c simultaneously.

Let us now look at some possible simplifications in calculations for nonstationary processes in thermogenerators. From a determination of the temperature distribution over the layers of the thermogenerator heat-transfer surface (Fig. 3), we can draw the natural conclusion that the greatest temperature differential occurs across the layer of semiconductor material and electrical insulation.

To make a quantitative estimate, we introduce the parameter $\Delta T/b$, where ΔT is the temperature difference across the given layer, while b is the layer thickness. For $T_{\text{med}} = 100$°C and $T_c = 20$°C, for the specific type of thermogenerator considered (Fig. 1b) we have:

for layer 1, $\Delta T/b_1 = 0.6$ deg/mm;
for layer 2, $\Delta T/b_2 = 80$ deg/mm;
for layer 3, $\Delta T/b_3 = 0.6$ deg/mm;
for layer 4, $\Delta T/b_4 = 28$ deg/mm.

As a consequence, we can neglect the influence of layers 1 and 3 with no loss of accuracy.

In such case, the equation system for the nonstationary processes in the thermogenerator can be simplified, and written as

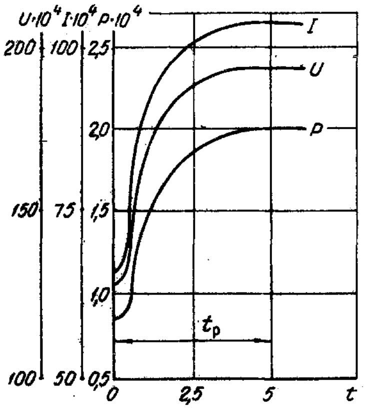

Fig. 2

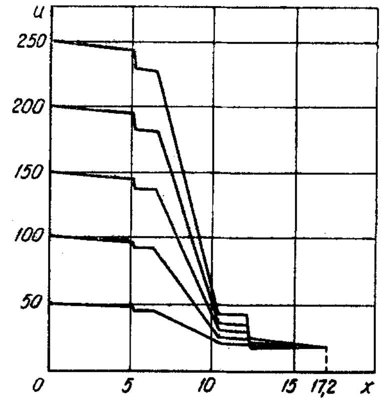

Fig. 3

Fig. 2. Current (I, *a*), voltage (U, v), and power (P, W) of thermogenerator as function of time (t, sec) under abrupt change in T_{med} from 100 to 150°C.

Fig. 3. Static distribution of temperature (u, °C) over layer thickness (x, mm) in thermogenerator.

$$\frac{\partial u_1}{\partial t} = X_1^2 \frac{\partial^2 u_1}{\partial x^2} - \frac{1}{2}\vartheta_J, \qquad x_0 < x < x_1;$$
$$\frac{\partial u_2}{\partial t} = X_2^2 \frac{\partial^2 u_2}{\partial x^2}, \qquad x_1 < x < x_2; \tag{13}$$
$$\frac{\partial u_3}{\partial t} = X_3^2 \frac{\partial^2 u_3}{\partial x^2} + \frac{1}{2}\vartheta_J, \qquad x_2 < x < x_3.$$

Here the initial conditions are

$$u_j(x)|_{t=0} = \varphi_j(x) \quad (j = 1, 2, 3), \tag{14}$$

the boundary conditions are

$$\left(\frac{\partial u_1}{\partial x} + h_1 u_1\right)_{x=x_0} = h_1 T_{\text{med}}$$
$$\left(-\frac{\partial u_3}{\partial x} + h_3 u_3\right)_{x=x_3} = h_3 T_c \tag{15}$$

and the layer contact conditions are

$$\lambda_{T_1} \frac{\partial u_1}{\partial x} = \lambda_{T_2} \frac{\partial u_2}{\partial x} + q_{\text{ph}}, \; x = x_1;$$
$$u_1 = u_2;$$
$$\lambda_{T_2} \frac{\partial u_2}{\partial x} = \lambda_{T_3} \frac{\partial u_3}{\partial x} + q_{\text{pc}}, \quad x = x_2;$$
$$u_2 = u_3,$$

where $u_j = u(x, t)$ $(j = 1, 2, 3)$ is a continuous function of the temperature distribution, having piecewise-continuous derivatives up to order two, respectively.

NOTATION

u is the temperature distribution function;
F is the cross-sectional area;
γ is the specific gravity of the material;
V is the volume of the material;
G is the flow rate of the heat-transport medium (coolant);
δ is the density of the material;
c is the heat capacity of the material;

X^2	is the thermal diffusivity;
α_T	is the heat-transfer coefficient;
α_s	is the thermal emf coefficient;
λ_T	is the thermal-conductivity coefficient;
T_h	is the temperature of the thermocouple hot junctions;
T_c	is the temperature of the thermocouple cold junctions;
T_{med}	is the temperature of the heat-transport medium;
T_c	is the temperature of the coolant;
ϑ_J	is the per-second temperature variation caused by Joule heat;
q_{ph}	is the density of the heat flux resulting from the Peltier heat absorbed at the thermocouple hot junctions;
q_{pc}	is the density of the heat flux produced by the Peltier heat liberated at the thermocouple cold junctions;
$h = \alpha_T/\lambda_T$	is a coefficient;
R	is the electrical resistance of the load;
r	is the internal electrical resistance of a single thermocouple;
$M = r/k_{ser}r$	is the optimal ratio of electrical resistances;
k_{ser}	is the number of series-connected thermocouples;
b	is the thickness of the layer of material;
t	is the time;
I	is the current;
U	is the voltage;
P	is the electrical power.

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